# The Arm of God 

# Geometric and Algebraic Metaphors for the Sh'ma and the Unity of God 

Hear O Israel, the Lord is Our God, The Lord is One

(Echod Ado-noy ,Elo-haynu Ado-noy ,Israel Sh'ma)

NOTES: Read the Hebrew words and English phonetic transliteration, word-by-word, from Right to Left. $\leftarrow$
To avoid casual use of G-d's names, Jewish practice pronounces
"Ado-noy" ("Lord") as HaShem ("The Name") and "Elo-him" ("G-d") as Elokim.

## Introduction

As those who have followed the work of the Meru Foundation know, we have attempted to understand how the shape of the special Tefillin HAND (made of leather ribbon) that forms the letters of the alphabet was known and defined in traditional sources.


Left and Right HANDS shown holding Meru Foundation's specially shaped Tefillin with 22-Shadowgrams of the Tefillin showing the Hebrew letters strung between them

When I began this paper I assumed that no algebraic function was involved because I thought that modern algebraic functions were not known in the ancient world. It seemed that only a geometric definition or a craftsperson's operational definition would have been available. Further investigation showed that this is not necessarily so. While it is true that our modern algebraic notation was not available in the ancient world and while ancient methods of calculation may have been different from ours today, there is ample evidence of extensive algebraic knowledge in the ancient world. Certain algebraic functions were well known and tables of values used to find solutions were available.
As we will see, the reciprocal function is of special interest. Egyptian mathematical documents show lists of whole numbers and their reciprocals.

From the jacket cover of Mathematics in the Time of the Pharaohs by Richard Gillings, 1972, MIT Press (bold emphasis added):
"The mathematical operations used were extremely limited in number but were adaptable to a great many applications. The Egyptian number system was decimal, with digits sequentially arranged (much like our own, but reading right to left), allowing them to add and subtract with ease. They could muliply any number by two, and to accomplish more extended multiplications made use of a binary process, successively multiplying results by two and adding those partial products that led to the correct final result. Division was done in a similar way. They could fully manipulate fractions, even though all of them (with one exception) were expressed in the unwieldy form of sums of unit fractions - those having " 1 " as their numerator. ..."
"The range of mathematical problems that were solved using these limited operational means is far wider than many historians of mathematics acknowledge. Gillings gives examples showing that the Egyptians were able, for example, to solve problems in direct and inverse proportion; to evaluate certain square roots; to introduce the concept of a "harmonic mean" between two numbers; to solve linear equations of the first degree, and two simultaneous equations, one of the second degree; to find the sum of terms of arithmetic and geometric progressions; to calculate the area of a circle and of cylindrical (possibly even spherical) surfaces; . . ."

Professor Gillings gives many examples of the use of algebraic functions, including the inverse function that is required for the equation we will present in this paper, in the text of his book.

Therefore，even though the philosophical，cultural and spiritual definition must have been primary and the geometric／mechanical means adopted to fulfill the spiritual requirements must have been secondary，the means of modeling the Tefillin HAND could also have been understood both geometrically and algebraically．This is what we propose to investigate here．

The most obvious place to look for a philosophically and spiritually appropriate definition of Tefillin，and the means of binding or tying of the leather ribbons which hold the Tefillin on our arm and forehead，is in what the sages of the Talmud tell us about Tefillin and in the Sh＇ma，the credo of Judaism，in which Tefillin are discussed and their use described．So this is where we will look．

Although there are several different traditions that describe how the leather ribbons attached to Tefillin are knotted， wound and bound on the arm，hand，and forehead，all agree that we should be able to see certain Hebrew letters，usually including Shin＂ש，Dalet $\boldsymbol{T}$ Final Mem $\boldsymbol{\square}$ ，and Yod＂，in the shape of，or in knots in，the leather ribbon that binds Tefillin to our hand，arm，and forehead．Meru Foundation＇s findings demonstrate how all of the Hebrew letters may be seen when the Tefillin strap is specially shaped（as described in detail previously．）


The Meru Foundation Tefillin（Left）is not the same as the representative traditional example shown（Right），but the ＂alphabet effect＂is similar．The Meru Tefillin shows all of the letters in different gestures of the Hand while the example of traditional Tefillin shows 3－letters，Shin שט，Dalet 7，Yod＂spelling Shad－dai，the God name alluding to the dispensation of spiritual energy into the world，fixed on the Hand．

## THE SH＇MA

The basic premise of the Sh＇ma（which specifies the use of Tefillin）and the reason why it is called the＂credo of Judaism＂ is stated in the opening line：

Hear O Israel，the Lord is Our God，The Lord is One
ール゙
（Echod Ado－noy ，Elo－haynu Ado－noy ，Israel Sh＇ma）
Read the Hebrew words and English phonetic transliteration，word－by－word，from Right to Left．$\leftarrow$
The meaning is clear：the two basic names and aspects of Divinity，represent only One Self－same Unity．
There is no doubt that God and the different aspects of God were known by many different names and in a wide range of contexts and cultural traditions in the ancient world．
－Many faiths knew that God was in the world．
－Many other faiths knew that meditation，dreams and prophesy originate from God within．

God is found as the source of and permeating all of the consensus cosmos AND God is found in the personal conscious experience of each of us. It was the unique discovery attributed to Abraham, the "founder" of the Abrahamic faiths, that the God of the Singular personal experience of meditational Unity, HaShem, is identical to the God of the panoply of All-There-Is in the cosmos, Elokim.


## The UNITY of Chochma INSIDE and Binah OUTSIDE

These two, seemingly different, aspects of God as they were known in the ancient world are, for the first time, identified as One-Unity. (See our essay, The God of Abraham, in Torus, The Journal of the Meru Foundation, Vol. 2, No. 3 for further details.) If we can interpret this most basic spiritual teaching in a geometric and/or a modern algebraic context, this might lead us to the path (or geometric shape) of the process that defines the Unity of HaShem and Elokim. This should be the same path or shape that generates the letters of the alphabet because the alphabet is also said to uniquely connect inner wisdom, chochma, with outer understanding, binah.

From Rabbi Aryeh Kaplan, Sefer Yetzirah: The Book of Creation, in Theory and Practice, (Copyright 1990, page 27, Samuel Weiser, ISBN 0-87728-690-6; parenthesis and emphasis added)
> "The letters are said to pertain to Foundation (Yesod), since it is only through the letters that Wisdom and Understanding can come together and be coupled. As discussed earlier, Wisdom is pure non-verbal thought. Understanding, on the other hand, can only be verbal, since if an idea cannot be expressed verbally, it cannot be understood. The only link between nonverbal Wisdom (Chochma), and verbal Understanding (Binah), consists of the letters of the alphabet."

The letters must span INSIDE-OUTSIDE, HaShem-Elokim. Thus the Tefillin HAND is ideal because that is exactly what the Tefillin does, as described in the Sh'ma.


The Knot at the back of the Head Tefillin also forms letters

# Analysis and Proposed Solution <br> The HaShem Aspect 

AdiN $\boldsymbol{T} \mathbf{T N}$ means pedestal or base and the same letter sequence, $\boldsymbol{M}-\boldsymbol{-} \mathbf{N}$, is sometimes used to indicate the current pronunciation of the Tetragrammaton, The 4-Letter-Name of God, pronounced "A-Doh and then "NoY" *

Here the Tetragrammaton
or 4-Letter-Name of God is represented by a 10 -sphere-point Tetractys triangle Pedestal.


The Tetragrammaton Tetractys consists of 10-letters arranged with 4 -letters on each edge of the Triangle. The total numerical, Gematria, value for all the letters is 72 , which also relates this formation to the 72-Letter-Name of God.


In 3-dimensions the 10-sphere-point Tetractys triangle can also be assembled as a 10-sphere-point Tetrahedron. The Tetrahedron is the geometric archetype of a pedestal or a base.

The Singular, Highest-Reaching, triangular or tetrahedral Pedestal, aspect of HaShem can be modeled by an arrow (the triangle or tetrahedron is the arrowhead) pointing up and out of the origin - similar to the kabbalistic concept of the line, Qav קי, that meets the ZimZum-contraction at creation. Therefore we will model the HaShem aspect as an arrow, a pointer, and a radial vector, $\mathbf{r}$, that reaches out from the origin.


## The Elokim Aspect

The Whole Encompassing Expanse of the All-There-Is aspect of Elokim can be modeled as the angular span of an arc. Eleh $\boldsymbol{Z}$. Expanse of nature and the cosmos. In the Sefer Zohar (Book of Splendor, a principle kabbalistic work) a' is reversed to read "Me", " $\boldsymbol{\square}$, (who) as in He Who Creates All.

Thus the Elokim aspect of God can be understood as similar to the swept-expanse of an arc, $\vartheta$, on a circular or cylindrical coordinate system.
The Expanse of Elokim in the Cosmos


Elokim* Modeled as the Sweep and Expanse of $\vartheta$

# A Geometric Metaphor of the Unity of HaShem and Elokim: <br> The Unitary Reciprocal Spiral r $\vartheta=1$ 

The ONE-Seed

Is there a geometric form and an algebraic function that can illustrate the HaShem, $\mathbf{r}$, aspect and the Elokim, $\vartheta$, aspect of God as Unity?

In fact there is. The equation $\mathbf{r} \vartheta=1$, which defines the Unitary Reciprocal (or Hyperbolic) Spiral, expresses this relationship and it represents a most unusual and extraordinary spiral curve.

As we will see later, this spiral curve can be geometrically projected onto our model of Continuous Creation by a simple, meaningful, and well-defined construction.

Taken together in the equation $\mathbf{r} \vartheta=\mathbf{1}$, as in the Sh'ma, the HaShem aspect and the Elokim aspect of God is seen to be One Unitary Whole - exactly algebraically equal to Unity at all times.

The equation is an algebraic metaphor and the curve itself a geometric metaphor for the Unity of HaShem - Elokim, the 4-Letter and the 5 -Letter Names, as stated in the Sh'ma.

## The Unitary Reciprocal Spiral



The spiral curve represents the span between two dimensions. The curve approaches the Unitary Point at the origin of the coordinate system at its inner spiral end, and it approaches the Line $\mathrm{y}=1$ at the outer linear end. The span between a point and a line is one archetypal dimensional step.

If the coordinate system is taken to be cylindrical and in 3 -dimensions, the "point" at the origin of the circular coordinate system in 2-dimensions can be understood as a "line", the vertical $z$-axis perpendicular to the page. Thus, the curve can be understood to span between two infinite lines ( $z$-axis and $y=1$ in the $x y$-plane) that are at right angles and separated by the unit distance between the $z$-axis which goes through the origin and $y=1$. As $\mathbf{r}$ (or y) approaches infinity, the $x$-axis and the line $y=1$ (in the $x-y$ plane) can be understood to be only infinitesimally separated (compared to the infinite length of each line.) Seen this way, the curve spans between two orthogonal dimensions. It spirals out from being wound around one line (the $z$-axis) and it converges linearly into $y=1$ and $y=0$ (in the $x-y$ plane).

In both interpretations the transition is not just between a point and a line or between two orthogonal lines either crossing or at unit separation, but also between circular-spiraling and linear extension. Not only is the relationship between two adjacent dimensions "transcendental", but the relationship between circular and linear is also. This is the universal and ubiquitous mathematically transcendental $\mathrm{Pi}, \Pi$, relationship that always relates the circumference and area of a circle to its linear diameter and circular to linear motion.

Also in both interpretations the transition is between Inside and Outside. The spiral curve demonstrates a path that uniquely and most elegantly represents a model of Integrity, another name for Unity. The spiral unfurls from the origin and then it stands upright as it approaches the line $\mathrm{y}=1$. In the Talmud a person of integrity is said to be Toko K'Varo תובו כבתו. Toko K'Varo תוכו כבריו means "inside like outside" and it represents the idea of moral transparency. A person who is the same inside and outside acts without presenting a facade or a false persona. Such a person is essentially egoless and might qualify as a Tzaddik, a righteous person, a saint. Our spiral curve reaches between an inner point and an outer expanse. It is a path that a Tzaddik might take; it can represent "a hero's journey."

## From 2-Dimensional Flatland to Continuous Creation

When the Unitary Reciprocal Spiral is projected onto a cube-octahedral sphere (this is the outline or frame of Meru Foundation's Continuous Creation model), the line $y=1$ and the $x$-axis are projected as Great Circles that cross not at infinity, but at the Great Circle equator of the sphere. Thus the "infinite" end of the curve (in 2-dimensions) becomes finite and its length fixed. The inner end of the curve is set to be exactly $11 / 2$-turns (3-Pi Radians) from the "infinite" end so as to match the edge of the 7-color map when it is projected onto a 2 -torus.

2-TORUS showing the 7-Color Map with the 3-Turn Spiral
Vortex Edge that is formed only when all 7-regions are identical.


Here the Unitary Reciprocal Spiral is Projected onto a Spherical Coordinate System


When three Projections of $\mathbf{r} \vartheta=1$ on $a$ Cube-Octahedroid Sphere are spaced at $120^{\circ}$, the shaded area between any two, looking directly into the vortex, is the Tefillin HAND.


The Great Circle Projections of $y=1$ and $y=0$ ( $x$-axis) meet on the Equator-Great Circle
$r$ approaches $\infty$ as projected $r$ approaches the Equator-Great Circle

## The Tefillin Hand



## The Unit Angle

In order to properly draw the spiral, it is necessary to have a definition of a unit angle. The natural unit of angular measure in the Torah appears to be 1-month.

- A month is approximately 27-days (star) or 29-days (sun).
- The Cube-octahedron naturally divides a circle into 30-degree arcs.
- 1-Radian $\left(360^{\circ} / 2 \Pi=\right.$ approx. $\left.57^{\circ}\right)$ is our current natural unit of angular measure, so $1 / 2$ Radian, approx. $29^{\circ}$, is also a good candidate for a natural unit.

It is likely that 27 or 28 -days out of a 364 (13x28) day, 13-month lunar year, or 365 -day, 12-month solar year, was taken to correspond to a natural angle of $30^{\circ}, 1 / 12$ th of a circle of $360^{\circ}$.

CUBE-OCTAHEDRON
showing natural division of the circle into 30-degree arcs
(After R. Buckminster Fuller, Synergetics II.)


Spheres pack with maximum density in the form of a Cube-octahedron or "13-petaled Rose"
The 13th sphere is in the center of the surrounding 12-spheres
It is also interesting to note that not only is the lunar month approximately 27 -days, but so too is the solar month. The Sun's surface rotates every 27 -days at middle latitudes. This rotation can be seen from the earth with the help of a simple pin-hole viewer (similar to those we use when watching a solar eclipse) because sunspots are visible as they move with the solar surface. Additionally. maximum and minimum sunspot activity occurs over a 22 -year cycle. This activity was worth monitoring because it coincides with weather and crop cycles. Thus it was likely to have been known in the ancient world.

## The Tefillin HAND Shape

It is interesting to note that the opening line of the Sh'ma plus the following line consists of 12-words and 49-letters:

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Letters
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4 9
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12 is the number of spheres on the outside of the cube-octahedral sphere-pack (see previous page) and 49 is the length, in tetrahedra, of the arm Tefillin's ribbon HAND shape when it is defined as a 1/6th section of the 3,10 Torus Knot model of Continuous Creation. (See Torus, The Journal of the Meru Foundation, Vol.2, Nos.2;4.)


The full 3,10 Knot consists of 99-Tetrahedra, so each half and each HAND must be 49-Tetrahedra long (with the odd 50th-Tetrahedron on the tip of the Thumb.)

In Talmud (Kinnim, Bird's Nests) it is possible that the spiral may be known as the path a bird of prey takes when it rises from its nest in an oasis, spirals up on the air currents, and then flies out over the desert and, eventually, the horizon. More generally, falconry, and the attendant knowledge of birds, especially birds of prey, has been traditional in middle-eastern cultures throughout the millennia.

The bird of prey, A-ZeN $\xlongequal{4}$ \#, is designated at the top of the shushon flower when the letters of the alphabet are written out on it. (See The Meru Project document for details)


The circled letters above the Shushon Flower designates a bird of prey


Flight Path of a Bird of Prey

## THE SLING SHOT AND SHOT PUT

The Tefillin Hand is also the path of the stone in David's slingshot used to kill Goliath. The sling and stone are swung around once and then the stone is released to fly off - B'YaDo בידי, in (from) the hand - in a straight line. The slingshot is קכלע, and so is the curtain in the Temple. קכֹע also designates twisting or weaving - a clear allusion to both the $11 / 2$-turn twist of the wind-up of a slingshot throw and to the woven nature of the Torus knots from which the Hand shape is taken.


The Hammer Throw or Sling-Shot (Adopted from Fuller, Synergetics)
Spinning around the origin builds angular momentum.
When the shot is released the angular momentum built up during the "Wind-up" is transformed into linear momentum. Then the shot flies straight until it falls to earth.

## The Spiral in the Palm of Our Hands

There are 27-bones in the Human Hand:
8 -Carpal (in the base of the palm),
5-Metacarpal (1-for each finger within the palm), and
14-Phalanges (the finger and thumb bones themselves.)


Left: From Fujitsu Corp. advertisement 24feb'97
Right: From Wynn Kapit \& Lawrence Elson The Anatomy Coloring Book
Nine-Bones, 1 in the Thumb and 8 in the Palm, form the Tefillin Hand spiral shape

## Hearing

The Sh'ma tells us to "Hear ...", so it is natural for us to examine our ears. As can be seen in the illustrations below, the Tefillin strap HAND is shaped like the Human Ear, the human instrument of hearing, both inside and outside.


The Human Ear


Strange EAR on sculpture from Attica Greece, circa 1200 BCE Biblical Arche. Review Vol.26,\#4


The Cochlea of the Human Ear is shaped like an ear and both are shaped like the spiral of the Tefillin HAND. The Treble" Clef in musical notation has the same form.

## The Fourier Transform Model of HaShem-Elokim

In The God of Abraham (Torus, The Journal of the MERU Foundation, Vol.2, No.3) we presented a model, based on a mathematical operation known as the Fourier Transform, that demonstrated the Unique, Singular-Whole relationship of HaShem - Elokim:

As many musicians and electronics enthusiasts know, every musical pulse can be understood as the sum of many pure sine-wave tones. Similarly a square wave (called a square wave because, unlike the smoothly varying sine wave of a pure tone, a square wave changes abruptly and thus looks square- or rectangular-shaped) can be understood as consisting of the sum of many odd harmonics. Thus, by extrapolation, a truly infinite pulse can be understood as consisting of the sum of ALL possible pure tones - all sounding together (in phase) at the beginning of time.

THE FOURIER TRANSFORM


The way musicians understand the spectrum of musical tones and their harmonics is exactly the same as the procedure mathematicians call a Fourier Transform, where a sharp loud pulse (a finite stand-in for an infinite pulse) can be seen to consist of a broad spectrum of pure tones. Likewise an actual infinitely loud, sharp pulse - which we could compare to a musical Singularity - and which we can identify with the Infinite-Reaching, $r$ aspect, of HaShem, - would produce the harmonic spectrum of ALL tones - which we could liken to the ALL-THERE-IS aspect of Elokim.

TOKO K'VARO,
Outside
indicates a person of
INTEGRITY
Uprightness
<-UNITY - WHOLENESS->

The Expanse of Elokim in the Cosmos


DUTSIDE Understanding "Binah"
The Expanse of Elokim Is Like ALL-THERE-IS

The Fourier Transform of the Expanse of ALL-THERE-IS, is a Single, Pedestal-like, pulse of Infinite Intensity at the start of time - at creation - and, likewise, the Fourier Transform of the Expanse of an eternity of ALL-THERE-IS is a Single Infinite pulse.


The Singular Fulse at "CREATION" is the Fourier Transform of an eternity of ALL-THERE-IS; The Spectr um of ALL Tones and Harmonics, representing ALL-THERE-IS
is the Fourier Transform of the "CREATION" Pulse.
Notice that the UNIT amplitude of the Expanse of ALL-THERE-IS corresponds to the UNIT offset of $\mathrm{y}=1$ from the x axis. Like the Unit distance between $\mathrm{y}=1$ and the x -axis, when compared to the infinite extent and duration of ALL-THERE-IS, the Unit offset is negligible.
The Infinite, Upright, Pedestal-like, Pulse is perpendicular to ALL-THERE-IS. This is the same relationship as between the $z$-axis and the line $y=1$ (or the $x$-axis.) Thus the spiral $\mathbf{r} \vartheta=\mathbf{1}$ and the Fourier Transform are two different ways of illustrating essentially the same relationship, the Identity and Complementarity of HaShem and Elokim, as stated in the opening line of the Sh'ma.

# Miscellaneous Examples 

The spiral is also shaped like the Christian Shepherd's Crook and like the spiral beneath the Eye-of-Horus and the spiral sprouting from the head of some of the Egyptian hieroglyphic images.

Note: The Horus spiral = 1/32th: a month of days



Eye of Horus showing Spiral Curve

From NATURE'S HARMONIC UNITY: A Treatise on It's Relation to Proportional Form; Samuel Colman, N.A.; C. Arthur Coan, LL.B. ed; Benjamin Blom, Inc.; New York, 1971; LCC 78-177520, page 115:

> CHAPTER IX On Conchology


## Ionic Volute


#### Abstract

"Among all the enchanting things in the world of beauty, the shell is one of the most perfect as a representative form of spiral construction, and the one where the spiral is most obvious to the casual eye, at once announcing itself a geometric product. That the Greeks studied the shell from the standpoint of architect and designer may be proven by ample evidence; they were largely a seafaring race, and the rare shells their sailors brought back from distant lands must have attracted the attention of artists and students of beauty in those days as they have in many subsequent periods. That the shell is the source of the lonic volute is emphasized by the fact that in this volute the equiangular spiral ceases at the last turn where the curve passes into the horizontal lines of the capital, corresponding to the last turn in the spiral of many shells always equiangular to that point, as described in relation to the begonia, and as will presently be seen in the shell of the nautilus family."




Note how the relatively straight Back-bone or Spine of the Fetus spirals into its sphere-shaped Head just as the straight part of the Tefillin HAND spirals into the Head or origin of the coordinate system.

## Fish Swimming in Water

Note the shape of the Vortex Spirals


Left: Showing Transverse Motion and Spiral Form;


Right: Squirting Colored Dye displays a chain of Spiral Vortices
(From: "An Efficient Swimming Machine", by Triantafyllou and Triantafyllou, Scientific American, March 1995, p. 66.)

## A CLUSTER OF GRAPES in a KIDDUSH CUP



Right: Sphere by sphere outline of the "thumb"-spiral part of one Tefillin Hand model.
Notice that three similar spirals (Shown on the left in Red,Green, Yellow), separated by 120-degrees can also be fitted into the same sphere-pack without interference.


The three 7-Sphere straight sections follow the edges of square faces of the cube-octahedral sphere-pack
ON TEFILLIN


Square-shaped HEAD TEFILLIN shown worn between the eyes
"The mouth, the secret of creation in the "present" moment, whose "breath" represents the continual emanation of energymatter, is between the two eyes, as the present is between the past and the future. Regarding the position of the head tefillin, the Torah instructs that their inscribed words be "a memory between your eyes, so that the Torah of God shall be in your mouth." The word "memory" itself, in Hebrew, means "a source of speech (as in the phrase: "in that place that I will allow you to speak My Name I will come to you and bless you")."
From: R. Yitzchak Ginsburgh, The Hebrew Letters: Channels of Creative Consciousness, page 4. Copyright 1990 by Gal Einai Publications, P.O. Box 14132, Jerusalem, Israel.

## The Philippine Wine Dance

> Right hand path shown. The right hand path and left hand path are the same, but inverted. Photograph of 2-First Hands


Various views of The Philippine Wine Dance
Mathematically this is the Double-covering; It is related to the Dirac String Trick which describes the spin of Fermions with spin=1/2.

For further details on the Dirac String Trick, see The Dirac String Trick, First Hand.
When a right-handed (above) and a left-handed (below) pair of First Hands are placed thumb to fingers, the pair traces the path of the Philippine wine dancer's hands for each cycle.

## Examples from The Book of Kells



From: Sharkey, Color added for clarity

## Shang Dynasty Vessel



From: Huxley, Red color added to spirals for clarity

THE ARM OF GOD WORKING PAPER

## Examples from Leonardo Davinci's Notebooks


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