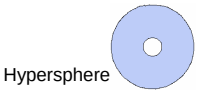
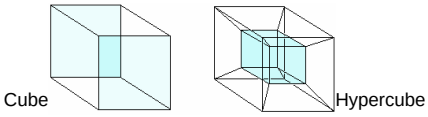
The Dirac String Trick - *First Hand*

by Stan Tenen

The *string* of the *Dirac String Trick* is often depicted as a floppy ribbon connecting the surface of an *out*-sphere to the surface of a concentric *in*-sphere. Together, the surface of the *in*-sphere / *out*-sphere represents the surface of a hypersphere. This is analogous to a tesseract which models a hypercube as an *in*-cube within an *out*-cube (with the two cube's corresponding vertices connected.)



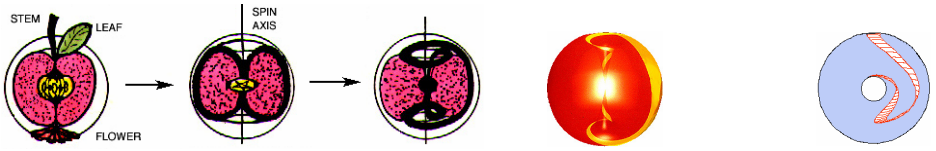
In-sphere/*Out*-sphere model



In-cube/*Out*-cube - Tesseract model

Biologically, an *in*-sphere within an *out*-sphere can model a seed-pit or ovary (an *in*-sphere) within its fruit (an *out*-sphere). Here the floppy ribbon represents a living tree that connects its seed / *in*-sphere to its fruit / *out*-sphere.

Various Expressions of a Model Fruit *Out*-Sphere with a Seed-Pit *In*-Sphere



Hypersphere modeled as a 2-Torus defined by a 3-turn Spiral Vortex

Vortex "Apple" graphic courtesy of James Fournier

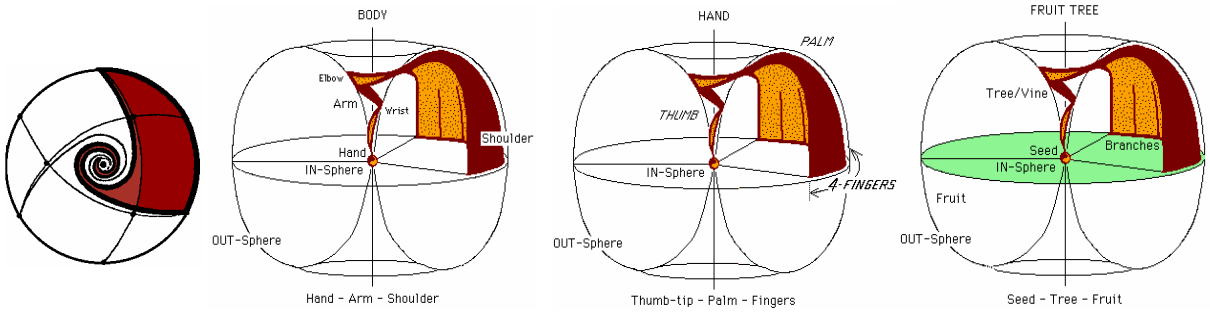
Dirac String Trick drawing courtesy of Louis Kauffman, U. Ill, Chicago[1]

Because the ribbon is flexible, it can be looped under the *in*-sphere and untwisted whenever the *in*-sphere is turned to twist it. Two turns of the *in*-sphere are untwisted by passing the ribbon under the *in*-sphere. The *in*-sphere can be turned continuously as long as the ribbon flops under it for every two turns. This *double-covering* geometry also helps physicists to visualize how fermions have spin of 1/2.

Dirac String Trick Storyboard, courtesy of Louis Kauffman, U. Ill, Chicago[1]

There is a beautiful computer simulation, *Air on Dirac Strings*, by Francis, Kauffman, and Sandin.[2], that demonstrates how the twistings of the Dirac String are the same as that of the arms of a person performing the Philippine wine dance.[3] It takes a rotation above and another rotation below the dancer's shoulder - a double covering - to untwist the dancer's arm while the dancer's palm, holding the wine glass level, turns continuously. As with the *in*-sphere, the dancer can continue to rotate their hands in the same direction indefinitely. The dancer's arm takes the place of the Dirac String; the dancer's shoulder is on the surface of the *out*-sphere; the dancer's palm holds the *in*-sphere - which holds the rotating wine-glass level (and keeps the wine from spilling) throughout each cycle.

FIRST HAND

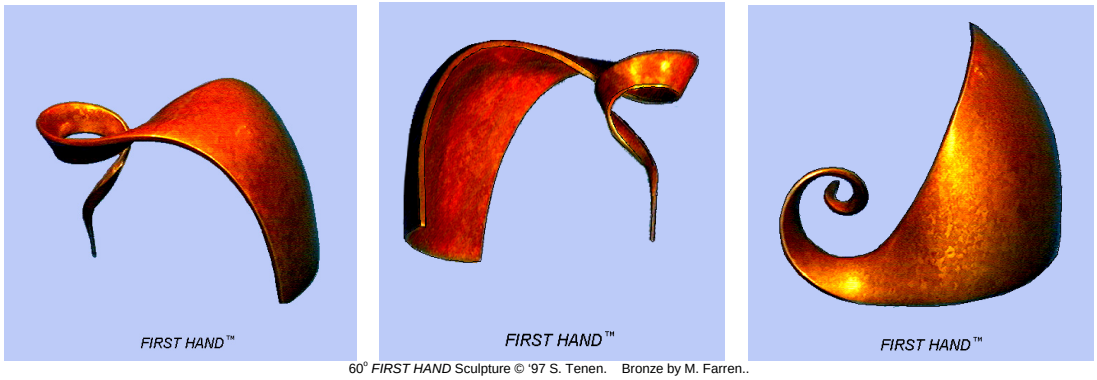


The *In*-sphere, Dirac String, and *out*-sphere as Represented by
A Hand, Arm, and Shoulder; A Thumb-tip, Palm, and Fingers; A Seed, Tree, and Fruit

The ***First Hand*** sculpture is an expression of this process. It is a section of the surface of a hypersphere, modeled as a 2-torus, in the form of an apple. It is an attempt to represent the projection of the seed, at the tip of the thumb, into the whole of the fruit cupped in palm of the hand.

This expression of nature's *gesture* from seed, via vine or tree, into fruit is also a model of a human hand. Here, the process of pointing - the gesture - takes the form of the pointer - our hand. What we will in our mind, we can point to in the world with our hand; what our hand holds can be seen in our mind's eye.

Metaphorically, the Dirac String links an *in*-sphere to an *out*-sphere in much the same way as our hand links our mind to our world, our palm links our thumb to our fingers, our arm links our hand to our body, and a tree links its seed to its fruit. ***First Hand*** is designed to express this metaphor.



60° *FIRST HAND* Sculpture © '97 S. Tenen. Bronze by M. Farren..

While we can only speculate on how the Philippine wine dance (also known by other names in various cultures throughout the world) was understood in the past, it has a truly remarkable property when it is expressed as ***First Hand***: Its various 2-dimensional views appear to be nearly identical to a particular alphabet of medieval Hebrew letters.

Various side views of

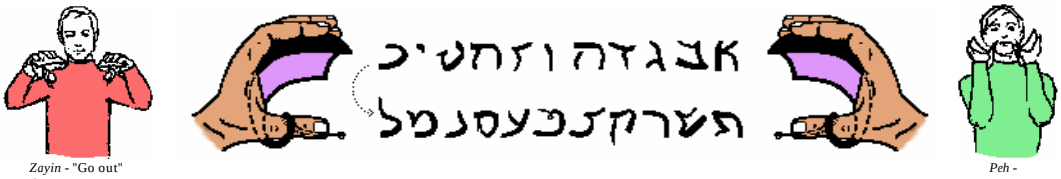
The Philippine Wine Dance

Right hand path shown. The right hand path and left hand path are the same, but inverted. Photograph of 2-*First Hands*

When a right-handed (above) and a left-handed (below) pair of ***First Hands*** are placed thumb to fingers, the pair traces the path of the Philippine wine dancer's hands for each cycle.

The Rashi-Nachmanides Rabbinic Script Alphabet

(Read the top line right to left and the bottom line left to right for alphabetical order.)



Zayin - "Go out" "mouth / speak" *Peh* -

Center: Each of the 22-letters of the Hebrew alphabet shown is a different 2-dimensional view of the same 3-dimensional *hand* sculpture. *Right, Left:* Sample hands and gestures

When various gestures are made while wearing ***First Hand***, different letters are seen by the wearer. In Hebrew each letter has a name and a meaning. The gesture that displays each letter to the wearer expresses the meaning of the name of the Hebrew letter; words spelled with these letter gestures can often be correctly read by naive viewers.

References:

1. *Knots and Physics*, by Louis H. Kauffman, World Scientific, 1991,'94, pp. 419-442, SO(3) and the Belt Trick; p. 441, graphic.
2. Videotape *Air on Dirac Strings*, by Francis, Kauffman, Sandin, U. Ill. Chicago, Electronic Visualization Laboratory,1993. For additional information contact Daniel J. Sandin, Director, Electronic Visualization Laboratory (M/C 154), University of Illinois at Chicago, 851 S. Morgan St. Rm 1120 SEO, Chicago, IL 60607-7053. dan@evl.eecs.uic.edu
3. See *A Topological Picturebook*, Geo. K. Francis, Springer-Verlag,1987, pages 134-5: "Dirac used it [the Plate Trick, *Figure* 5.] to illustrate how the Lie group SO(3) of rotations in 3-space is doubly covered by the group S_3 of unit length quaternions. Note that the latter is the 3-sphere in 4-space."